Exercise 8

Strontium-90 has a half-life of 28 days.

(a) A sample has a mass of 50 mg initially. Find a formula for the mass remaining after t days.

(b) Find the mass remaining after 40 days.

(c) How long does it take the sample to decay to a mass of 2 mg?

(d) Sketch the graph of the mass function.

Solution

Part (a)

Assume that the rate of mass decay is proportional to the amount of mass remaining at any given time.

$$\frac{dm}{dt} \propto -m$$

There's a minus sign here because mass is being lost as time increases. Change the proportionality to an equation by introducing a (positive) constant k.

$$\frac{dm}{dt} = -km$$

Divide both sides by m.

$$\frac{1}{m}\frac{dm}{dt} = -k$$

Rewrite the left side by using the chain rule.

$$\frac{d}{dt}\ln m = -k$$

The function you have to differentiate to get $-k$ is $-kt + C$, where C is any constant.

$$\ln m = -kt + C$$

Exponentiate both sides.

$$e^{\ln m} = e^{-kt+C}$$

$$m(t) = e^C e^{-kt}$$

Use a new constant m_0 for e^C.

$$m(t) = m_0 e^{-kt} \tag{1}$$

Use the fact that strontium-90 has a half-life of 28 days to get k.

$$\frac{m_0}{2} = m_0 e^{-k(28)}$$

$$\frac{1}{2} = e^{-28k}$$

$$\ln\frac{1}{2} = \ln e^{-28k}$$

$$-\ln 2 = -28k \ln e$$

$$k = \frac{\ln 2}{28} \approx 0.0247553 \text{ day}^{-1}$$

Equation (1) then becomes

$$\begin{aligned} m(t) &= m_0 e^{-\left(\frac{\ln 2}{28}\right)t} \\ &= m_0 e^{\ln 2^{-t/28}} \\ &= m_0 (2)^{-t/28}. \end{aligned}$$

Use the fact that the mass is 50 milligrams initially to determine m_0.

$$m(0) = m_0(2)^{-(0)/28} = 50 \quad \to \quad m_0 = 50$$

Therefore, the mass in milligrams after t days have passed is

$$m(t) = 50(2)^{-t/28}.$$

Part (b)

The mass remaining after 40 days is

$$m(40) = 50(2)^{-40/28} \approx 18.5749 \text{ mg}.$$

Part (c)

To find how long it takes the sample to decay to 2 mg, set $m(t) = 2$ and solve the equation for t.

$$m(t) = 2$$

$$50(2)^{-t/28} = 2$$

$$2^{-t/28} = \frac{1}{25}$$

$$\ln 2^{-t/28} = \ln \frac{1}{25}$$

$$\left(-\frac{t}{28}\right) \ln 2 = -\ln 25$$

$$t = \frac{28 \ln 25}{\ln 2} \approx 130.028 \text{ days}$$

Part (d)

Below is a graph of the mass versus time.